

This article was published as a part of the Data Science Blogathon. The understanding shape of data is a crucial action. It helps to understand where the most information is lying and analyze the outliers in a given data. In this article, we'll learn about the shape of data, the importance of skewness, and kurtosis. The types of skewness and kurtosis and Analyze the shape of data in the given dataset. Skewness may violate model assumptions or may reduce the interpretation of feature importance. In statistics, skewness is a degree of asymmetry observed in a probability distribution that deviates from the symmetrical normal distribution (bell curve) in a given set of data. The normal distribution, data symmetrically distributed. The symmetrical distribution has zero skewness as all measures of a central tendency lies in the middle. When data is symmetrically distributed. the left-hand side, and right-hand side, contain the same number of observations.). But, what if not symmetrical distributed? That data is called asymmetrical data, and that time skewness comes into the picture. Types of skewness 1 Positive skewed or right-skewed In statistics, a positively skewed data, the measures of the central tendency (mean, median, and mode) equal each other, with positively skewed data, the measures of the central tendency (mean, median, and mode) equal each other, with positively skewed data, the measures are dispersing, which means Positively Skewed Distribution is a synce of distribution where the mean, median, and mode of the data is greater than the median (a large number of data-pushed on the right-hand side). In other words, the results are bent towards the lower side. The mean will be more than the median as the median is the middle value and mode is always the highest value The extreme positive skewness is not desirable for distribution, as a high level of skewness can cause misleading results. The data transformation is the log transformation. The log transformation proposes the calculations of the natural logarithm for each value in the dataset. 2. Negatively skewed distribution. In statistics, negatively skewed distribution model where more values are plots on the right side of the graph, and the tail of the distribution is spreading on the left-hand side). Negatively skewed, the mean of the data is less than the median (a large number of data-pushed on the left-hand side). Negatively skewed the mean of the data is less than the median (a large number of data-pushed on the left-hand side). positive or zero. Median is the middle value, and mode is the highest value, and due to unbalanced distribution median will be higher than the mean. Calculate the skewness Subtract a mode from a mean, then divides the difference by standard deviation. As Pearson's correlation coefficient differs from -1 (perfect negative linear relationship) to +1 (perfect positive linear relationship), including a value of 0 indicating no linear relationship), when we divide the covariance values by the standard deviation, it truly scales the value down to a limited range of -1 to +1. That accurately the range of the correlation values. Pearson's first coefficient of skewness is helping if the data present high mode. But, if the data present high mode or various modes, Pearson's second coefficient is not preferred, and Pearson's second coefficient is not preferred, and Pearson's second coefficient may be superior, as it does not rely on the mode. But, if the data present high mode or various modes, Pearson's second coefficient of skewness is helping if the data present high mode. But, if the data present high mode. But, if the data present high mode or various modes, Pearson's second coefficient is not preferred, and Pearson's second coefficient is not preferred. deviation. If the skewness is between -0.5 & 0.5, the data are extremely skewed) or greater than -1 (positive skewed), the data are extremely skewed. If the skewness is lower than -1 (positive skewed) or greater than 1 (positive skewed) or between 0.5 & 0.5, the data are extremely skewed. Kurtosis refers to the degree of presence of outliers in the distribution. In finance, kurtosis is a statistical measure, whether the data is heavy-tailed or light-tailed in a normal distribution. In finance, kurtosis is associated with a high level of risk for an investment because it indicates that there are high probabilities of extremely large and extremely small returns. On the other hand, a small kurtosis signals a moderate level of risk because the probabilities of extreme returns are relatively low. The excess kurtosis is used in statistics and probability theory to compare the kurtosis coefficient with that normal distribution. Excess kurtosis is used in statistics and probability theory to compare the kurtosis coefficient with that normal distribution. distribution), negative (Platykurtic distribution), or near to zero (Mesokurtic distribution). Since normal distributions have a kurtosis of 3, excess kurtosis is calculating by subtracting kurtosis by 3. Excess kurtosis = Kurt - 3 Types of excess kurtosis Leptokurtic or heavy-tailed distribution (kurtosis more than normal distribution). Mesokurtic (kurtosis same as the normal distribution). Leptokurtic or short-tailed distribution). Leptokurtic (kurtosis > 3) Leptokurtic (kurtosis indicate that distribution). Leptokurtic or short-tailed distribution). positive kurtosis indicates a distribution where more of the numbers are located in the tails of the data points are present in high proximity with mean. A platykurtic distribution is flatter (less peaked) when compared with the normal distribution. Mesokurtic (kurtosis = 3) Mesokurtic is the same as the normal distribution, which means kurtosis is near to 0. In Mesokurtic, distributions are moderate in breadth, and curves are a medium peaked height. tailed or light-tailed in a normal distribution. Data can be positive-skewed (data-pushed towards the right side) or negative-skewed (data-pushed towards the right side). When data skewed, the tail region may behave as an outlier for the statistical model, and outliers unsympathetically affect the model's performance especially regression-based models. Some statistical models are hardy to outliers like Tree-based models, but it will limit the possibility to try other models. So there is a necessity to transform the skewed data to close enough to a Normal distribution), negative (Platykurtic distribution), or near to zero (Mesokurtic distribution). Leptokurtic distribution (kurtosis more than normal distribution). Mesokurtic distribution). Mesokurtic distribution). The media shown in this article on skewness and Kurtosis are not owned by Analytics Vidhya and is used at the Author's discretion. Related Copyright © 2008-2022 by Stan Brown, BrownMath.com Technology: Contents: The first thing you usually notice about a distribution's shape is whether it has one mode (peak) or more than one. If it's unimodal (has just one peak), like most data sets, the next thing you notice is whether it's symmetric or skewed to one side. If the bulk of the data is at the left and the right tail is longer, we say that the distribution is skewed right or positively skewed; if the peak is toward the right and the left tail is longer, we say that the distribution is skewed right or positively skewed; if the peak is toward the right and the left tail is longer, we say that the distribution is one of the many skewed distributions that are used in mathematical modeling. Beta( $\alpha$ =4.5,  $\beta$ =2) skewness = -0.5370 1.3846 - Beta( $\alpha$ =4.5,  $\beta$ =2) skewness = +0.5370 The first one is moderately skewed right: its right tail is longer and most of the distribution is at the left. You can get a general impression of skewness by drawing a histogram (MATH200A part 1), but there are also some favor one, some favor one, some favor another. This Web page presents one of them. In fact, these are the same formulas that Excel uses in its "Descriptive Statistics" tool in Analysis Toolpak, and in the SKEW() function. You may remember that the mean and standard deviation have the same units. However, the skewness has no units: it's a pure number, like a z-score. Computing The moment coefficient of skewness of a data set is skewness: q1 = m3 / m23/2 (1) where  $m3 = \sum (x-\bar{x})^2 / n$  and  $m2 = \sum (x-\bar{x})^2 / n$  is the mean and n is the sample size, as usual. m3 is called the third moment of the data set. m2 is the mean and n is the sample size, as usual. m3 is called the third moment of the data set. m2 is the mean and n is the sample size, as usual. m3 is called the third moment of the data set. m2 is the mean and n is the sample size, as usual. m3 is called the third moment of the data set. m2 is the mean and n is the sample size, as usual. m3 is called the third moment of the data set. m2 is the mean and n is the sample size, as usual. m3 is called the third moment of the data set. m2 is the mean and n is the sample size, as usual. m3 is called the third moment of the data set. m2 is the mean and n is the sample size, as usual. m3 is called the third moment of the data set. m2 is the mean and n is the sample size, as usual. m3 is called the third moment of the data set. m2 is the mean and n is the sample size, as usual. m3 is called the third moment of the data set. m2 is the mean and n is the sample size, as usual. m3 is called the third moment of the data set. m2 is the mean and n is the sample size, as usual. m3 is called the third moment of the sample size, as usual. m3 is called the third moment of the sample size, as usual. m3 is called the third moment of the sample size, as usual. m3 is called the third moment of the sample size, as usual. m3 is called the third moment of the sample size, as usual. m3 is called the third moment of the sample size, as usual. m3 is called the third moment of the sample size, as usual. m3 is called the third moment of the sample size, as usual. m3 is called the third moment of the sample size, as usual. m3 is called the third moment of the sample size, as usual. m3 is called the third moment of the sample size, as usual. m3 is called the third moment of the sample size, as usual. m3 is called the third moment of the sample size, as usual. m3 is called the third moment of the sample size, course the average value of z is always zero, but what about the average of z3? Suppose you have a few points far to the right of the mean. Since cubing the deviations to the right of the mean. You'll remember that you have to compute the variance and standard deviation slightly differently, depending on whether you have the sample. The same is true of skewness. If you have the sample, you need the sample skewness: (2) sample skewness: (The formula comes from Joanes and Gill 1998 [full citation in "References", below].) Excel doesn't concern itself with whether you have a sample or a population: its measure of skewness is always G1, the sample skewness. Example 1: College Men's Height(inches)ClassMark, xFre-quency, f 59.5-62.5615 62.5-62.5615 65.56418 65.5-68.56742 68.5-71.57027 71.5-74.5738 Here are grouped data for heights of 100 randomly selected male students, adapted from Spiegel and Stephens (1999, 68) [full citation in "References", below]. A histogram shows that the data are skewed left, not symmetric. But how highly skewed are they, compared to other data sets? To answer this question, you have to compute the skewness. Begin with the sample mean. (The sample mean. (The sample mean in hand, you can be the skewness) + 100  $\bar{x} = 6745 \div 100 = 67.45$  Now, with the mean in hand, you can compute the skewness. (Of course in real life you'd probably use Excel or a statistics package, but it's good to know where the numbers come from.) Class Mark, xFrequency, fxf  $(x-\bar{x})^2$ f  $(1-\bar{x})^2$ f  $(1-\bar{x$  $6745n/a852.75-269.33 \bar{x}$ , m2, m3 67.45n/a8.5275-2.6933 Finally, the skewness is q1 = m3 / m23/2 = -2.6933 / 8.52753/2 = -0.1082 But wait, there's more! That would be the skewness if you had data for the whole population. But obviously there are more than 100 male students in the world, or even in almost any school, so what you have here is a sample, not the population. You must compute the sample skewness: =  $[\sqrt{100 \times 99} / 98] [-2.6933 / 8.52753/2] = -0.1098$  Interpreting If skewness is positive, the data are positively skewed or skewed left, meaning that the left tail is longer. If skewness = 0, the data are perfectly symmetrical. But a skewness of exactly zero is quite unlikely for real-world data, so how can you interpret the skewness number? There's no one agreed interpret the skewness of exactly zero is quite unlikely for real-world data, so how can you interpret the skewness of exactly zero is quite unlikely for real-world data, so how can you interpret the skewness of exactly zero is quite unlikely for real-world data. is less than -1 or greater than +1, the distribution can be called moderately skewed. If skewness is between  $-\frac{1}{2}$  and  $+\frac{1}{2}$ , the distribution can be called approximately symmetric. With a skewness of -0.1098, the sample data for student heights are approximately symmetric. Caution: This is an interpretation of the data you actually have. When you have data for the whole population, that's fine. But when you have a sample skewness, can you conclude anything about the population skewness? To answer that question, see the next section. Inferring Your data set is just one sample drawn from a population. Maybe, from ordinary sample is skewed even though the population. Maybe, from ordinary sample is skewed even though the population. there is skewness in the population. But what do I mean by "too much for random chance to be the explanation"? To answer that, you need to divide the sample skewness (SES) to get the test statistic; Zg1 = G1/SES where This formula is adapted from page 85 of Cramer (1997) [full citation in "References", below]. (Some authors suggest  $\sqrt{6/n}$ , but for small samples that's a poor approximately 2. (This is a two-tailed test of skewness  $\neq 0$  at roughly the 0.05 significance level.) If Zq1 < -2, the population is very likely skewed negatively (though you don't know by how much). If Zq1 > 2, the population is very likely skewed negatively (though you don't know by how much). positively (though you don't know by how much). Don't mix up the meanings of this test statistic and the amount of skewness. The amount of skewness tells you whether the whole population is probably skewed, but not by how much: the bigger the skew. the number, the higher the probability. Estimating GraphPad suggests a confidence interval for skewness: (4) 95% confidence interval of population skewness is an unbiased estimator of population skewness for normal distributions, but not others. So I would say, compute that confidence interval, but take it with several grains of salt — and the further the sample skewness was G1 = -0.1098. The sample size was n = 100 and therefore the standard error of skewness is SES =  $\sqrt{(600 \times 99)} / (98 \times 101 \times 103) = 0.2414$  The test statistic is Zg1 = G1/SES = -0.1098 / 0.2414 = -0.45 This is between -2 and +2 (see above), so from this sample it's impossible to say whether the population is symmetric or skewed. Since the sample it's impossible to say whether the population is symmetric or skewed.  $-0.1098 \pm 2 \times 0.2414 = -0.1098 \pm 0.4828 = -0.5926$  to +0.3730. You can give a 95% confidence interval of skewness as about -0.59 to +0.37, more or less. Kurtosis Because this article helps you, please click to donate! Because this article helps you, please click to donate! Because this article helps you, please click to donate! Because this article helps you, please click to donate at BrownMath.com/donate. The other common measure of shape is called the kurtosis As skewness involves the third moment of the distribution, kurtosis involves the fourth moment. The outliers in a sample, therefore, have even more effect on the kurtosis, unlike skewness where they offset each other. You may remember that the mean and standard deviation have the same units as the original data, and the variance has the square of those units. However, the kurtosis, like skewness, has no units: it's a pure number, like a z-score. Traditionally, kurtosis has been explained in terms of the central peak. You'll see statements like this one: Higher values indicate a higher, sharper peak; lower values indicate a lower, less distinct peak. Balanda and MacGillivray (1988) [full citation in "References", below] has mention the tails: increasing kurtosis is associated with the "movement of probability mass from the shoulders of a distribution into its center and tails." been on a bit of a crusade to change this perception, and I think he makes a good case. We might say, following Wikipedia's article on kurtosis means more of the variance is the result of infrequent extreme deviations, as opposed to frequent modestly sized deviations." In other words, it's the tails that mostly account for kurtosis, not the central peak. The reference standard is a normal distribution, which has a kurtosis of 3. In token of this, often the excess kurtosis is simply kurtosis. A normal distribution has kurtosis exactly 3 (excess kurtosis) a contral distribution has kurtosis of 3. In token of this, often the excess kurtosis is simply kurtosis. kurtosis exactly 0). Any distribution with kurtosis ~3 (excess ~0) is called mesokurtic. A distribution with kurtosis 0) is called leptokurtic. Compared to a normal distribution, its tails are longer and fatter, and often its central peak is higher and sharper. Note that word "often" in describing changes in the central peak due to changes in the tails. Westfall 2014 [full citation in "References", below] gives several illustrations, suggested by Wikipedia, should help. All three of these distributions have mean of 0, standard deviation of 1, and skewness of 0, and all are plotted on the same horizontal and vertical scale. Look at the progression from left to right, as kurtosis increases. The normal distribution will probably be the subject of roughly the second half of your course; the logistic distribution is another one used in mathematical modeling. Don't worry at this stage about what these distributions mean; they're just handy examples that illustrate what I want to illustrate. Uniform(min= $-\sqrt{3}$ , max= $\sqrt{3}$ ) kurtosis = 1.2 Normal( $\mu$ =0,  $\sigma$ =1) kurtosis = 3, excess = 0.2 Sister ( $\alpha$ =0,  $\beta$ =0.55153) kurtosis = 4.2, excess = 0.2 Sister ( $\alpha$ =0,  $\beta$ =0.55153) kurtosis = 4.2, excess = 0.2 Sister ( $\alpha$ =0,  $\beta$ =0.55153) kurtosis = 4.2, excess = 0.2 Sister ( $\alpha$ =0,  $\beta$ =0.55153) kurtosis = 4.2, excess = 0.2 Sister ( $\alpha$ =0,  $\beta$ =0.55153) kurtosis = 4.2, excess = 0.2 Sister ( $\alpha$ =0,  $\beta$ =0.55153) kurtosis = 4.2, excess = 0.2 Sister ( $\alpha$ =0,  $\beta$ =0.55153) kurtosis = 4.2, excess = 0.2 Sister ( $\alpha$ =0,  $\beta$ =0.55153) kurtosis = 4.2, excess = 0.2 Sister ( $\alpha$ =0,  $\beta$ =0.55153) kurtosis = 4.2, excess = 0.2 Sister ( $\alpha$ =0,  $\beta$ =0.55153) kurtosis = 4.2, excess = 0.2 Sister ( $\alpha$ =0,  $\beta$ =0.55153) kurtosis = 4.2, excess = 0.2 Sister ( $\alpha$ =0,  $\beta$ =0.55153) kurtosis = 4.2, excess = 0.2 Sister ( $\alpha$ =0,  $\beta$ =0.55153) kurtosis = 4.2, excess = 0.2 Sister ( $\alpha$ =0,  $\beta$ =0.55153) kurtosis = 4.2, excess = 0.2 Sister ( $\alpha$ =0,  $\beta$ =0.55153) kurtosis = 4.2, excess = 0.2 Sister ( $\alpha$ =0,  $\beta$ =0.55153) kurtosis = 4.2, excess = 0.2 Sister ( $\alpha$ =0,  $\beta$ =0.55153) kurtosis = 4.2, excess = 0.2 Sister ( $\alpha$ =0,  $\beta$ =0.55153) kurtosis = 4.2, excess = 0.2 Sister ( $\alpha$ =0,  $\beta$ =0.55153) kurtosis = 4.2, excess = 0.2 Sister ( $\alpha$ =0,  $\beta$ =0.55153) kurtosis = 4.2, excess = 0.2 Sister ( $\alpha$ =0,  $\beta$ =0.55153) kurtosis = 4.2, excess = 0.2 Sister ( $\alpha$ =0,  $\beta$ =0.55153) kurtosis = 4.2, excess = 0.2 Sister ( $\alpha$ =0,  $\beta$ =0.55153) kurtosis = 4.2, excess = 0.2 Sister ( $\alpha$ =0,  $\beta$ =0.55153) kurtosis = 4.2, excess = 0.2 Sister ( $\alpha$ =0,  $\beta$ =0.55153) kurtosis = 4.2, excess = 0.2 Sister ( $\alpha$ =0,  $\beta$ =0.55153) kurtosis = 4.2, excess = 0.2 Sister ( $\alpha$ =0,  $\beta$ =0.55153) kurtosis = 4.2, excess = 0.2 Sister ( $\alpha$ =0,  $\beta$ =0.55153) kurtosis = 4.2, excess = 0.2 Sister ( $\alpha$ =0,  $\beta$ =0.55153) kurtosis = 4.2, excess = 0.2 Sister ( $\alpha$ =0,  $\beta$ =0.55153) kurtosis = 4.2, excess = 0.2 Sister ( $\alpha$ =0,  $\beta$ =0.55153) kurtosis = 4.2, excess = 0.2 Sister ( $\alpha$ =0,  $\beta$ =0.55153) kurtosis = 4.2, excess = 0.2 Sister ( $\alpha$ =0,  $\beta$ =0.55153) kurtosis = 4.2, excess = 0.2 Sister ( $\alpha$ =0,  $\beta$ =0.5515 and the tails. In other words, the intermediate values have become less likely and the central and extreme values have become more likely. The kurtosis increases while the standard deviation stays the same, because more of the variation is due to extreme values. continues. There is even less in the shoulders and even more in the tails, and the central peak is higher and narrower. How far can this go? What are the smallest possible kurtosis = 1, excess = -2 Student's t (df=4) kurtosis =  $\infty$ , excess =  $\infty$  A discrete distribution with two equally likely outcomes, such as winning or losing on the flip of a coin, has the lowest possible kurtosis. It has no central peak and no real tails, and you could say that it's "all shoulder" — it's as platykurtic as a distribution can be. At the other extreme, Student's t distribution with four degrees of freedom has infinite kurtosis. A distribution can't be any more leptokurtic than this. You might want to look at Westfall's (2014 [full citation in "References", below]) Figure 2 for three quite different distributions with identical kurtosis. Computing The moment coefficient of kurtosis of a data set is computed almost the same way as the coefficient of skewness: just change the exponent 3 to 4 in the formulas: kurtosis:  $a^2 = a^4 - 3$  (5) where  $m^4 = \sum(x - \bar{x})^2 / n$  Again, the excess kurtosis is generally used because the excess kurtosis of a normal distribution is 0.  $\bar{x}$  is the mean and n is the sample size, as usual. m4 is called the fourth moment of the data set. m2 is the variance, the square of the standard deviation. The kurtosis can also be computed as a4 = the average value of z is always zero, but the avera either side of the mean than when you have a lot of small ones. Just as with variance, standard deviation, and skewness, the above is the final computation. But if you have to compute the sample excess kurtosis using this formula, which comes from Joanes and Gill [full citation in "References", below]: (6) sample excess kurtosis: Excel doesn't concern itself with whether you have a sample or a population: its measure of kurtosis in the KURT() function is always G2, the sample excess kurtosis. Example: Let's continue with the example of the college men's heights, and compute the kurtosis of the data set.  $n = 100, \bar{x} = 67.45$  inches, and the variance m2 = 8.5275 in<sup>2</sup> were computed earlier. Class Mark, xFrequency, f x- $\bar{x}$  (x- $\bar{x}$ )4f 615-6.458653.84 6418-3.452550.05 6742-0.451.72 70272.551141.63 7385.557590.35  $\Sigma$  n/a19937.60 m4 n/a199.3760 Finally, the kurtosis is a4 = m4 / m2<sup>2</sup> = 199.3760/8.5275<sup>2</sup> = 2.7418 and the excess kurtosis is g2 = 2.7418-3 = -0.2582 But this is a sample, not the population, so you have to compute the sample excess kurtosis:  $G2 = [99/(98 \times 97)] [101 \times (-0.2582) + 6)] = -0.2091$  This sample is slightly platykurtic: its peak is just a bit shallower than the peak of a normal distribution. Inferring Your data set is just one sample drawn from a population. How far must the excess kurtosis be from 0, before you can say that the population also has nonzero excess kurtosis? The question is similar to the question is similar to the question about skewness, and the answers are similar to the question about skewness. kurtosis is from zero: (7) test statistic: Zg2 = G2 / SEK where The formula is adapted from page 89 of Cramer (1979) [full citation in "References", below]. (Some authors suggest  $\sqrt{24/n}$ , but for small samples that's a poor approximation. And anyway, we've all got calculators, so you may as well do it right.) The critical value of Zg2 is approximately 2 (This is a two-tailed test of excess kurtosis  $\neq 0$  at approximately the 0.05 significance level.) If Zg2 < -2, the population very likely has positive excess kurtosis (kurtosis >3, leptokurtic), though you don't know how much. For the sample college men's heights (n=100), you found excess kurtosis of G2 = -0.2091. The sample is platykurtic, but is this enough to let you say that the whole population is platykurtic (has lower kurtosis: SEK =  $2 \times SES \times \sqrt{(n^2-1)} / ((n-3)(n+5))$  n = 100, and the SES was previously computed as  $0.2414 \times \sqrt{(100^2-1)} / (n-3)(n+5)$  $(97 \times 105) = 0.4784$  The test statistic is Zg2 = G2/SEK = -0.2091 / 0.4784 = -0.44 You can't say whether the kurtosis of a normal distribution. Assessing Normality There are many ways to assess normality, and unfortunately none of them are without problems. One test is the D'Agostino-Pearson omnibus test (D'Agostino and Stephens [full citation in "References", below]). I've implemented the D'Agostino-Pearson test in an Excel workbook at Normality Check and Finding Outliers in Excel. It's called an omnibus test because it uses the test statistics for both skewness and kurtosis to come up with a single p-value assessing whether this data set's shape is too different from normal. The test statistic is (8) DP = Zq1<sup>2</sup> + Zq2<sup>2</sup> follows  $\gamma^2$  with df=2 You can look up the p-value in a table, or use  $\gamma^2$ cdf on a TI-83 or TI-84. This  $\gamma^2$  test always has 2 degrees of freedom, regardless of sample size. D'Agostino doesn't say why explicitly, but an author of one of the other chapters says that it was an empirical match, and that seems reasonable to me.  $\chi^2 cdf(2, 5.991464546) = 0.95$ , so if the test statistic is bigger than about 6 you would reject the hypothesis of normality at the 0.05 level. Caution: The D'Agostino-Pearson test has a tendency to err on the side of rejecting normality, particularly with small sample sizes. David Moriarty, in his StatCat utility, recommends that you don't use D'Agostino-Pearson for sample sizes below 20. For college students' heights you had test statistics Zg1 = -0.45 for skewness and Zg2 = 0.44 for kurtosis. The omnibus test statistic is  $DP = Zg1^2 + Zg2^2 = 0.45^2 + Zg2^2 + Zg2^2 + Zg2^2 = 0.45^2 + Zg2^2 + Zg$  $0.44^2 = 0.3961$  and the p-value for  $\chi^2(df=2) > 0.3961$ , from a table or a statistics calculator, is 0.8203. You cannot reject the assumption of normality. (Remember, you never accept the null hypothesis, so you can't say from this test that the distribution is normal.) The histogram suggests normality, and this test gives you no reason to reject that impression. Alternative Methods There's no One Right Way to test for normality. One of many alternatives to the D'Agostino-Pearson test is making a normal probability plot; the accompanying workbook does this. (See Technology near the top of this page.) TI calculator owners can use Normality Check on TI-83/84 or Normality Check on TI-89 See also: The University of Surrey has a good survey of problems with normality tests, at How do I test the normality of a variable's distribution? That page recommends using the test statistics Zg1 and Zg2 individually. Example 2: Size of Rat Litters For a second illustration of inferences about skewness and kurtosis of a population, I'll use an example from Bulmer [full citation at : Frequency distribution of litter size in rats, n=815 Litter size 123456 789101112 Frequency 73358116125126 1211075637254 I'll spare you the detailed calculations, but you should be able to verify them by following equation (1) and equation (2): n = 815,  $\bar{x} = 6.1252$ , m2 = 5.1721, m3 = 2.0316 skewness g1 = 0.1727 and sample skewness G1 = 0.1730 The standard error of skewness is SES =  $\sqrt{(6 \times 815 \times 814)} / (813 \times 816 \times 818) = 0.0856$  Dividing the skewness by the SES, you get the test statistic Zq1 = 0.1730 / 0.0856 = 2.02 Since this is greater than 2, you can say that there is some positive skewness in the population. Again, "some positive skewness. If you go on to compute a 95% confidence interval of skewness from equation (4), you get 0.1730±2×0.0856 = 0.00 to 0.34. What about the kurtosis? You should be able to follow equation (5) and compute a fourth moment of m4 = 67.3948. You already have m2 = 5.1721, and therefore kurtosis  $G2 = [814/(813 \times 812)]$  [ $816 \times (-0.4806 + 6) = -0.4762$  So the sample is moderately less peaked than a normal distribution. Again, this matches the histogram, where you can see the higher "shoulders". What if anything can you say about the population? For this you need equation (7). Begin by computing the standard error of kurtosis, using n = 815 and the previously computed SES of 0.0.0856: SEK = 2 × SES ×  $\sqrt{(n^2-1)} / ((n-3))$ (n+5)) SEK = 2 × 0.0856 ×  $\sqrt{(815^2-1)}$  /  $(812\times820)$  = 0.1711 and divide: Zg2 = G2/SEK = -0.4762 / 0.1711 = -2.78 Since Zg2 is comfortably below -2, you can say that the distribution of all litter sizes is platykurtic, less sharply peaked than the normal distribution. But be careful: you know that it is platykurtic, but you don't know by how much. You already know the population is not normal, but let's apply the D'Agostino-Pearson test anyway:  $DP = 2.02^2 + 2.78^2 = 11.8088$  p-value = P( $\chi^2(2) > 11.8088$  p) = 0.0027 The test agrees with the separate tests of skewness and kurtosis: sizes of rat litters, for the entire population of rats, are not normally distributed. References Balanda, Kevin P., and H. L. MacGillivray. 1988. "Kurtosis: A Critical Review". The American Statistician 42(2), 111-119. 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You can still get it via the Internet Wayback Machine. Spiegel, Murray R., and Larry J. Stephens. 1999. Theory and Problems of Statistics. 3d ed. McGraw-Hill. Westfall, Peter H. 2014. "Kurtosis as Peakedness, 1905–2014. R.I.P." The American Statistician 68(3): 191–195. Retrieved 2021-11-18 from What's New? 17 Mar 2022: In the interpretation of skewness level, changed "is" to "can be called", because there's nothing inevitable about Bulmer's terminology. 14-18 Nov 2021: Updated links here, here, and here. Located Öztuna 2006 at Archive.org, since it's no longer available at its original location. Also found the University of Surrey's How do I test the normality of a variable's distribution? at Archive.org, for the same reason. 26 Oct 2020: Converted page from HTML 4.01 to HTML5, and italicized variable names, and improved formatting of radicals. Added a note about the beta distribution and a similar note about the normal and logistic distributions. (intervening changes suppressed) 26-31 May 2010: Nearly a total rewrite. (intervening changes suppressed) 13 Dec 2008: New article. This page uses some material from my old Skewness and Kurtosis on the TI-83/84, which was first created 12 Jan 2008 and replaced 7 Dec 2008 by MATH200B Program part 1; but there are new examples and pictures and considerable new or rewritten material.

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